Carnegie Mellon University
Heinz College

94-775/95-865 Lecture 11: Image Analysis With Convolutional Neural Nets

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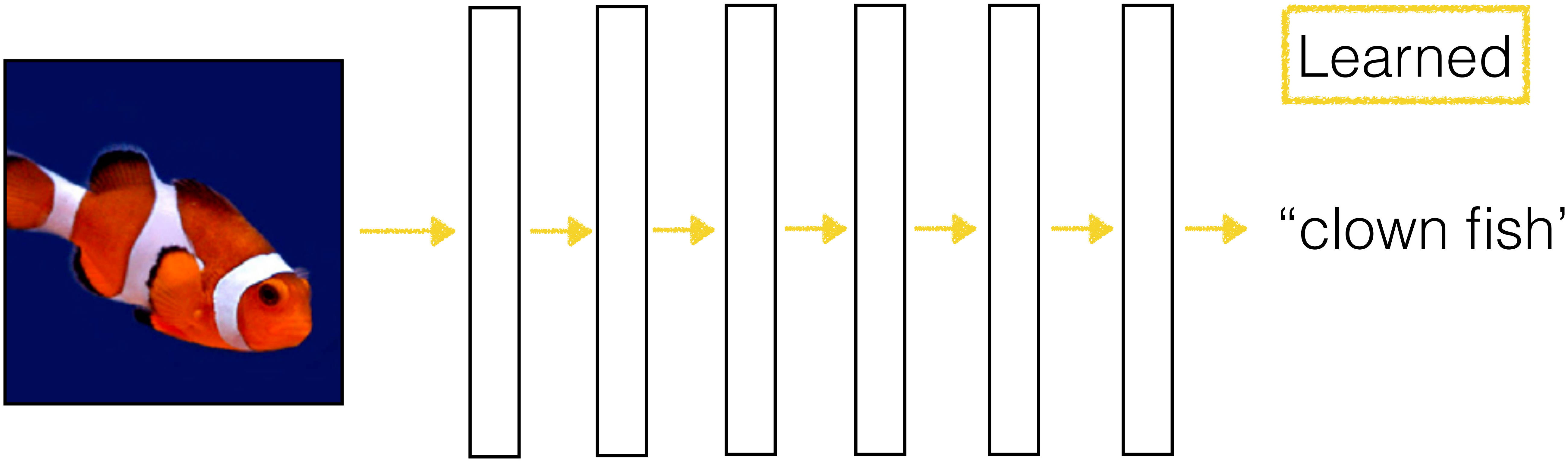
Deep Learning



- Inspired by biological neural nets *but otherwise not the same at all* (biological neural nets do *not* work like deep nets)
- Learns a layered representation
    - Tries to get rid of manual feature engineering
    - Need to design constraints for what features are learned to account for structure in data (e.g., images, text, …)

Learning a neural net amounts to curve fitting

We’re just estimating a function

Neural Net as Function Approximation

Given `input`, learn a computer program that computes `output`

this is a **function**

Single-layer neural net example:

```
def f(input):
  output = softmax(np.dot(input, W) + b)
  return output
```

the only things that we are learning
(we fix their dimensions in advance)

We are fixing what the function `f` looks like in code
and are only adjusting `W` and `b`!!!

Neural Net as Function Approximation

Given `input`, learn a computer program that computes `output`

Single-layer neural net example:

```
output = softmax(np.dot(input, W) + b)
```

Two-layer neural net example:

```
layer1_output = relu(np.dot(input, W1) + b1)

output = softmax(np.dot(layer1_output, W2) + b2)
```

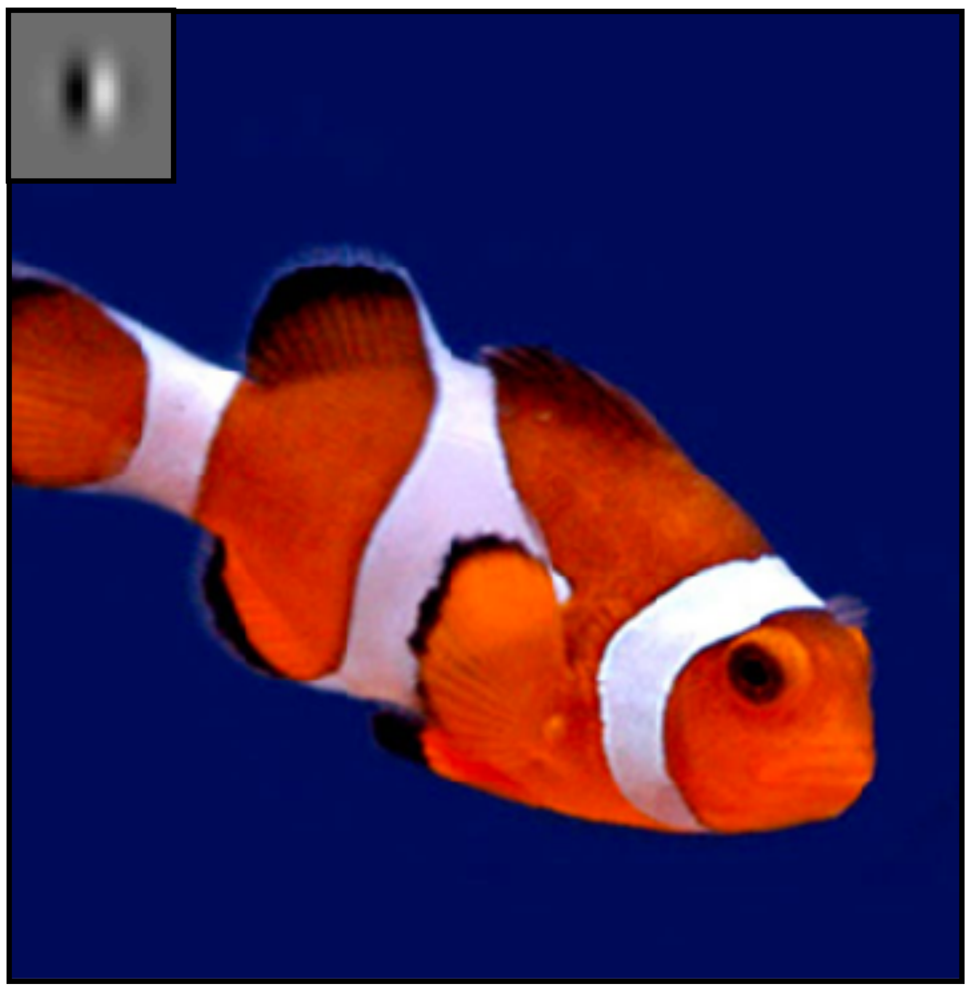
Learning a neural net: learning a simple computer program that maps inputs (raw feature vectors) to outputs (predictions)

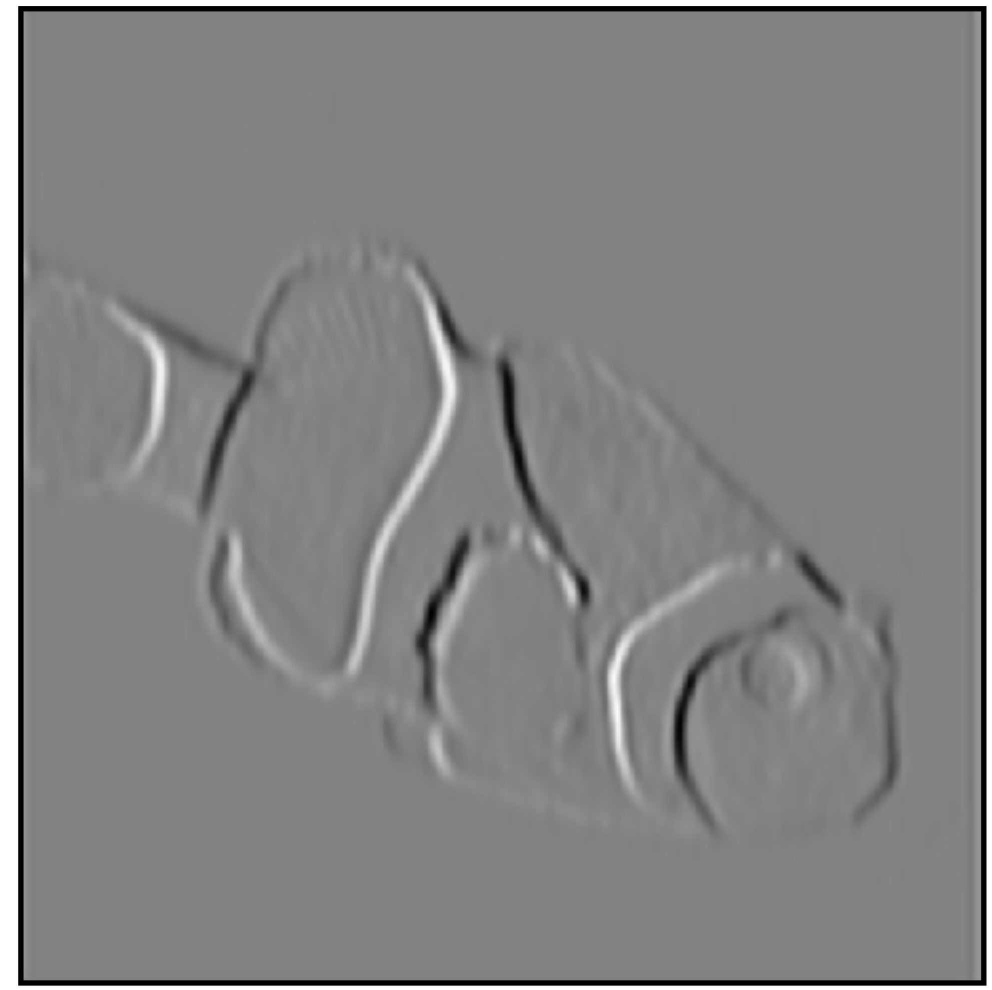
Architecting Neural Nets

- Increasing number of layers (depth) makes neural net more complex
  - Can approximate more functions
  - More parameters needed
    - More training data may be needed
- Designing neural net architectures is a bit of an art
  - How to select the number of neurons for intermediate layers?
  - Very common in practice: modify existing architectures that are known to work well (e.g., VGG-16 for computer vision/image processing)

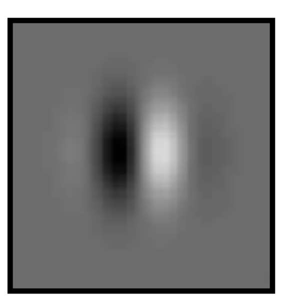
Image analysis with Convolutional Neural Nets (CNNs, also called convnets)

Convolution





filter



Slide by Phillip Isola

Convolution

0	0	0	0	0	0	0
0	0	1	1	1	0	0
0	1	1	1	1	1	0
0	1	1	1	0	0	0
0	1	1	1	1	1	0
0	0	1	1	1	0	0
0	0	0	0	0	0	0

Input image

0	0	0
0	1	0
0	0	0

Filter
(also called "kernel")

Convolution

0	0	0	0	0	0	0
0	0	1	1	1	0	0
0	1	1	1	1	1	0
0	1	1	1	0	0	0
0	1	1	1	1	1	0
0	0	1	1	1	0	0
0	0	0	0	0	0	0

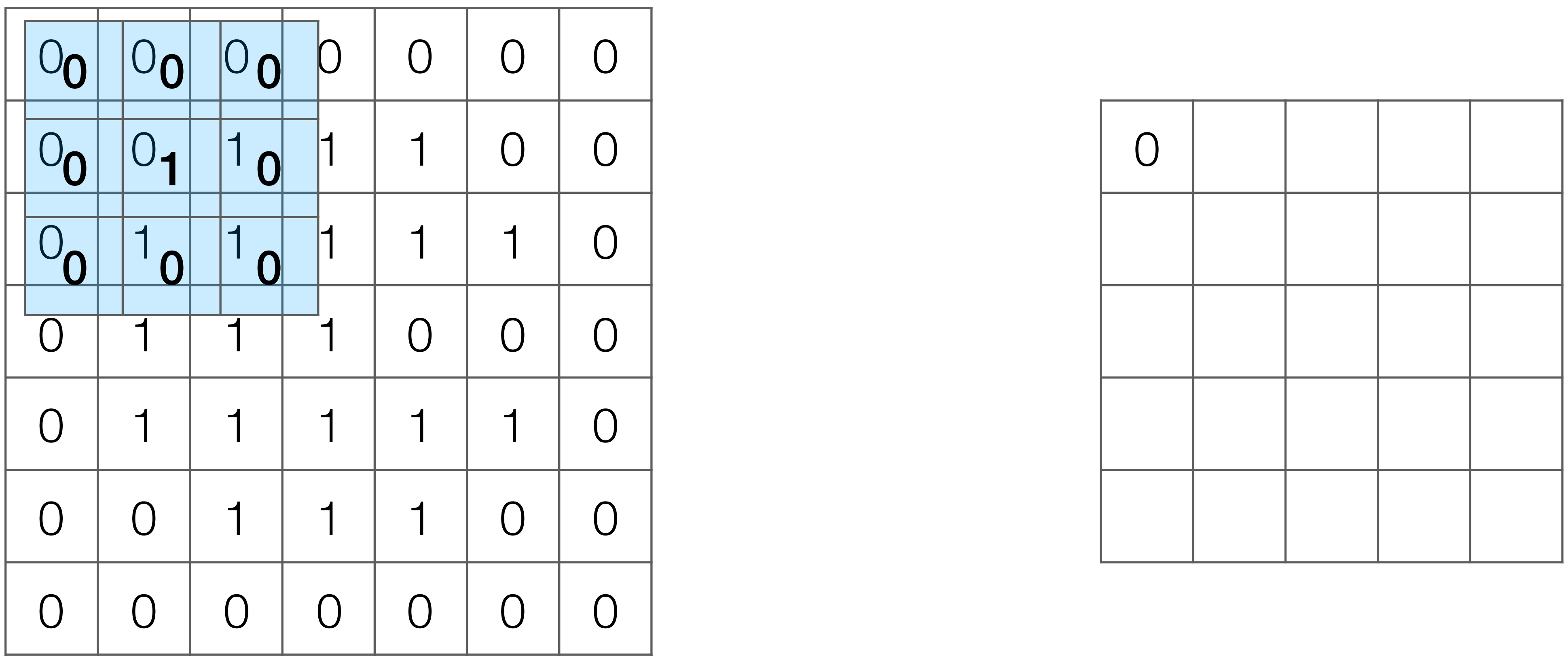
Input image

0	0	0
0	1	0
0	0	0

Filter
(also called “kernel”)

Convolution

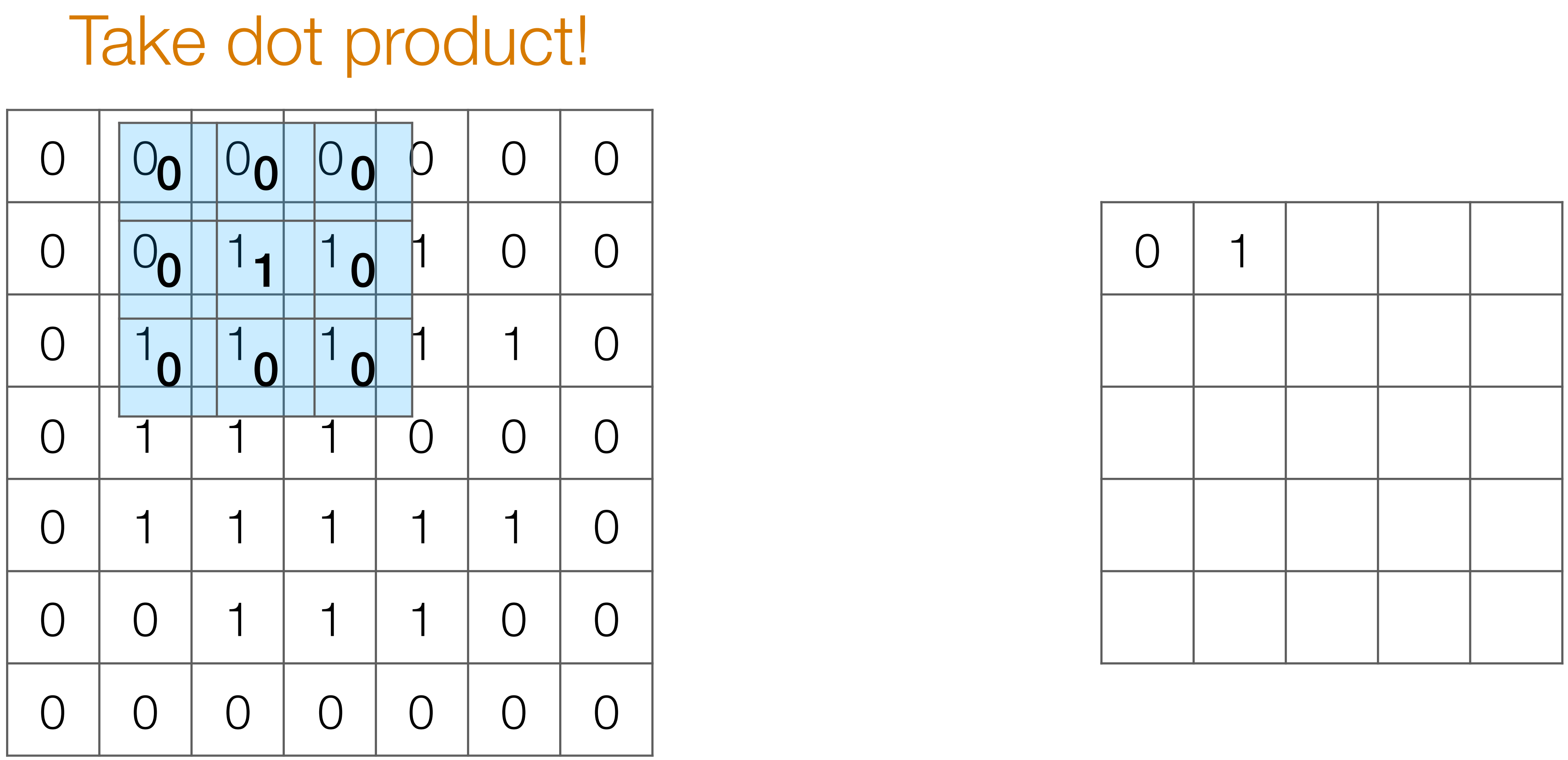
Take dot product!



Input image

Output image

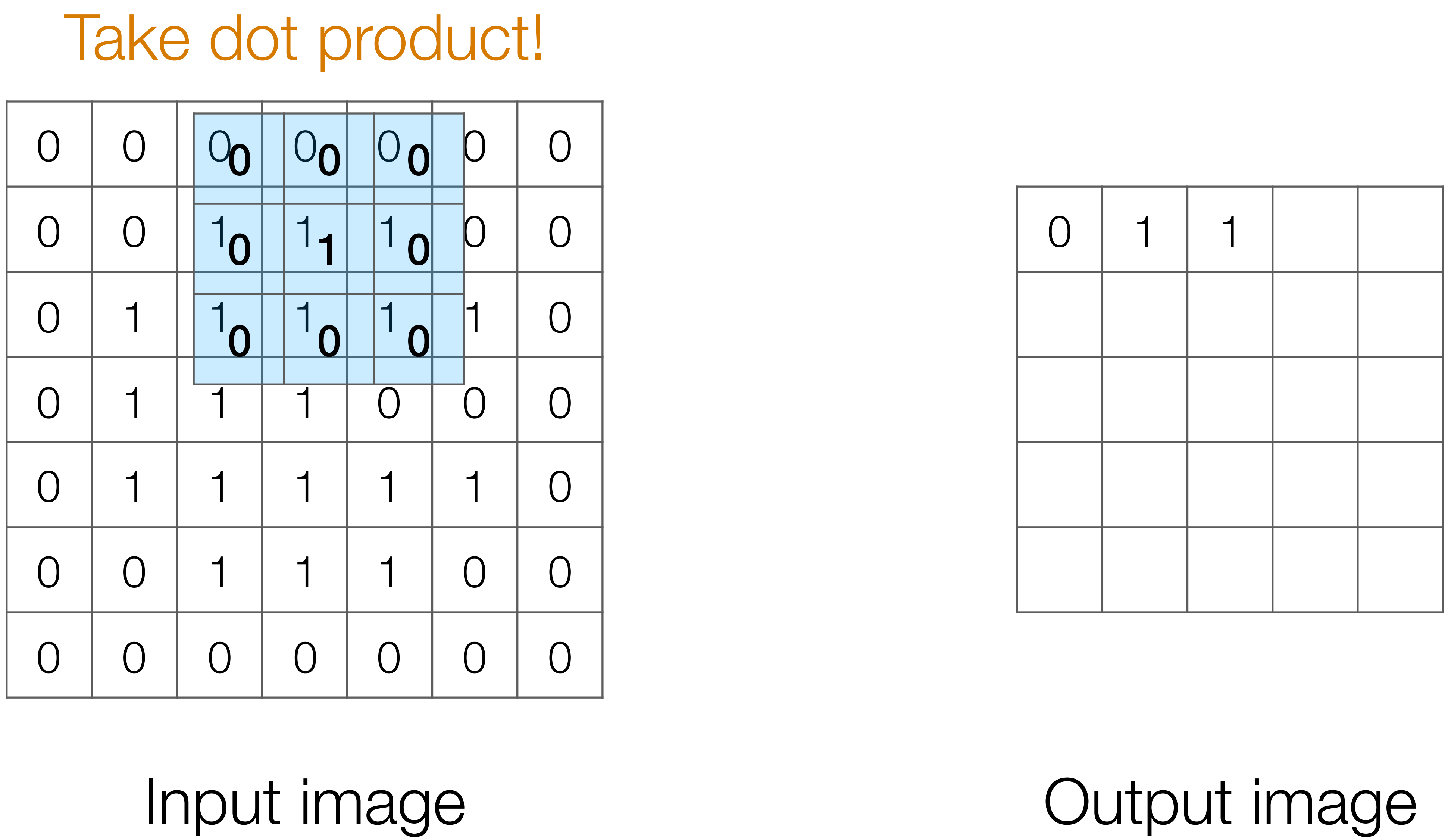
Convolution



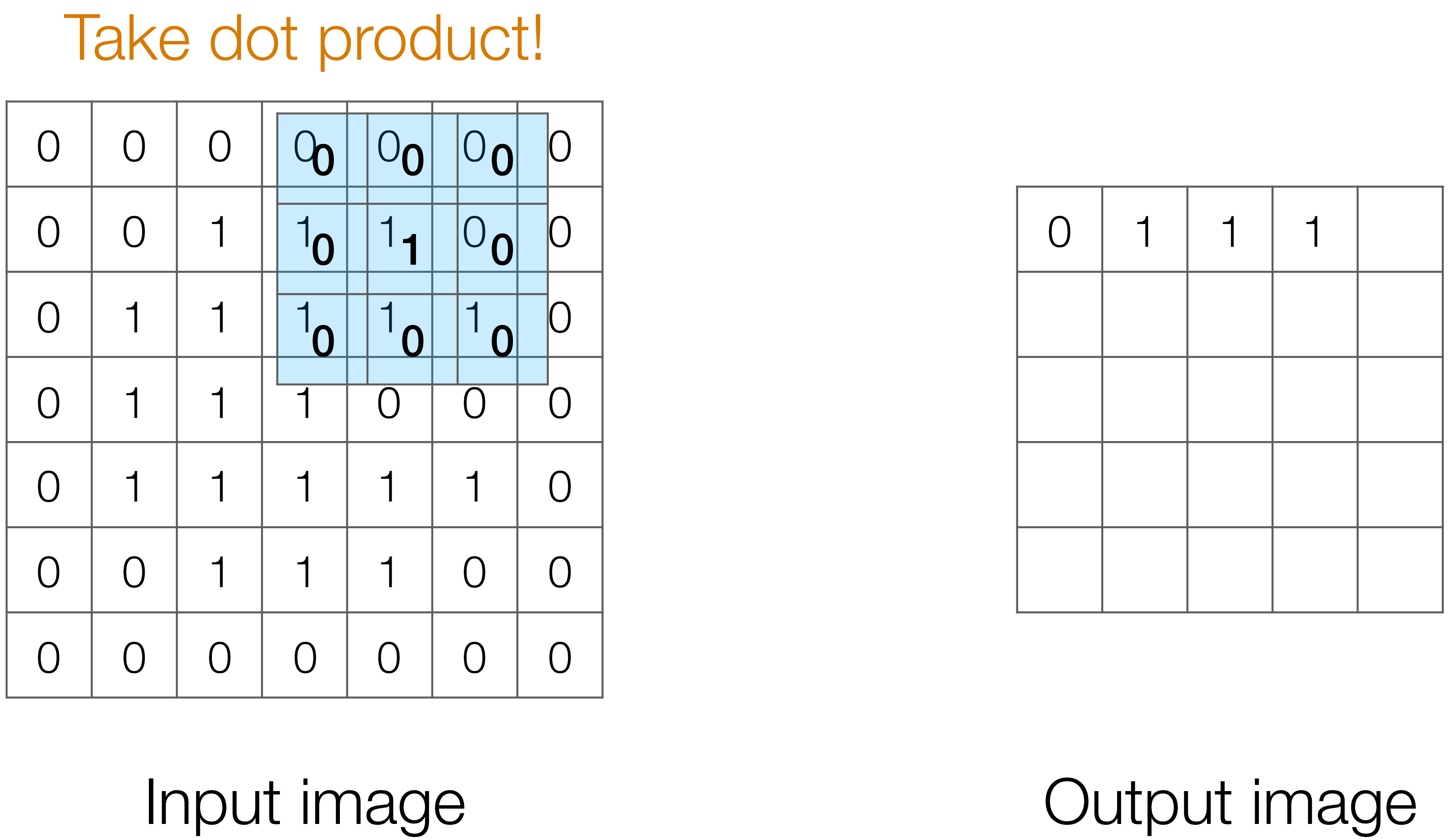
Input image

Output image

Convolution

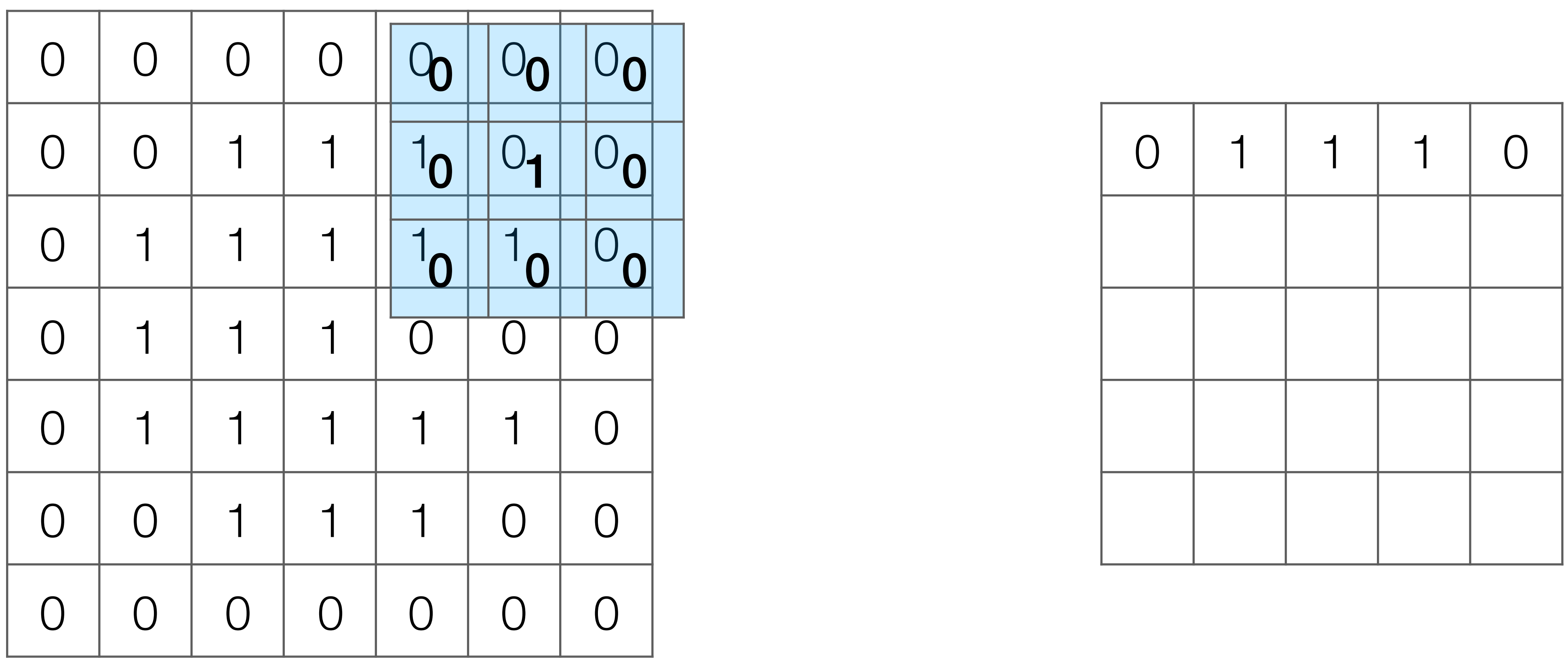


Convolution



Convolution

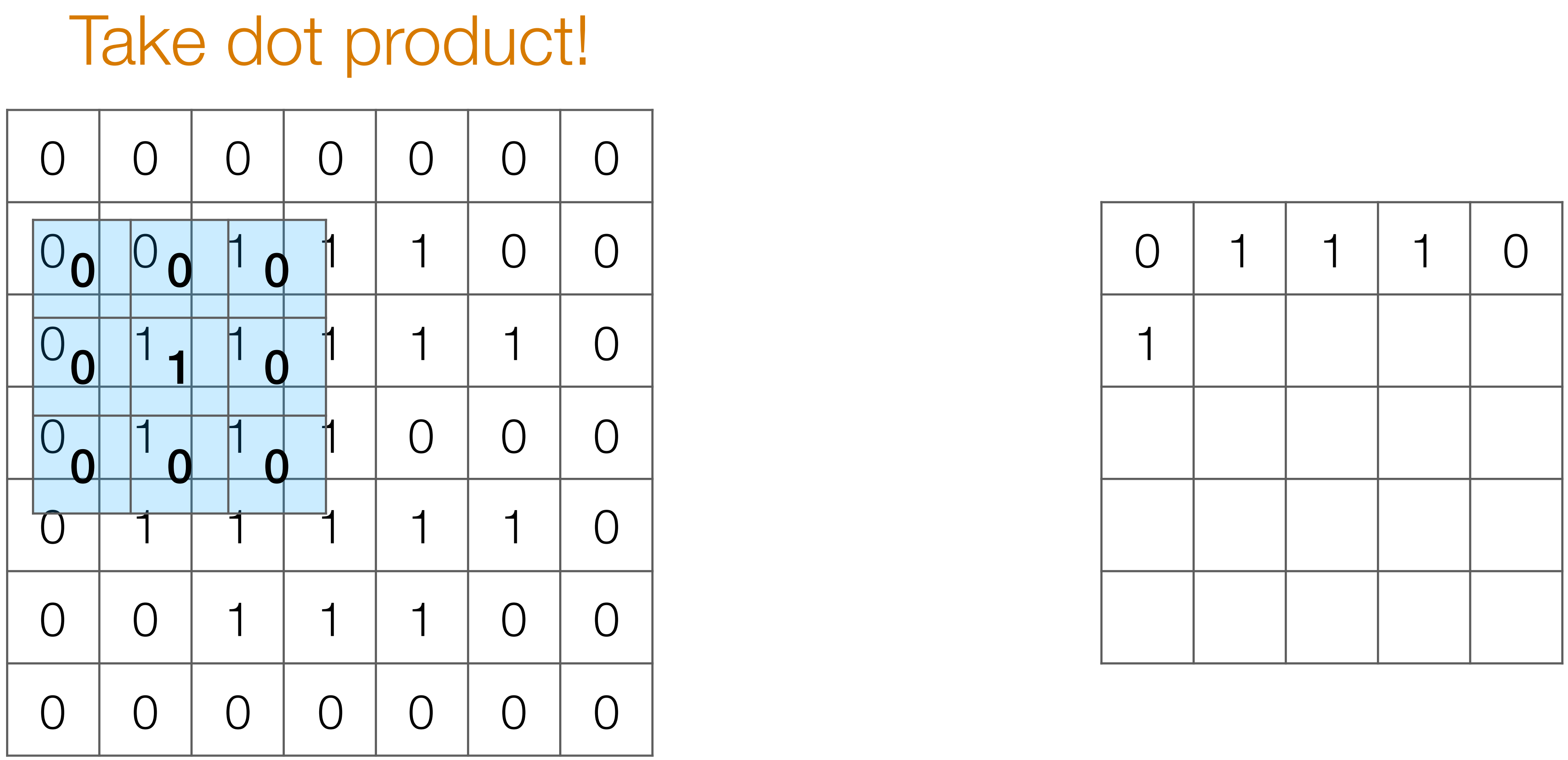
Take dot product!



Input image

Output image

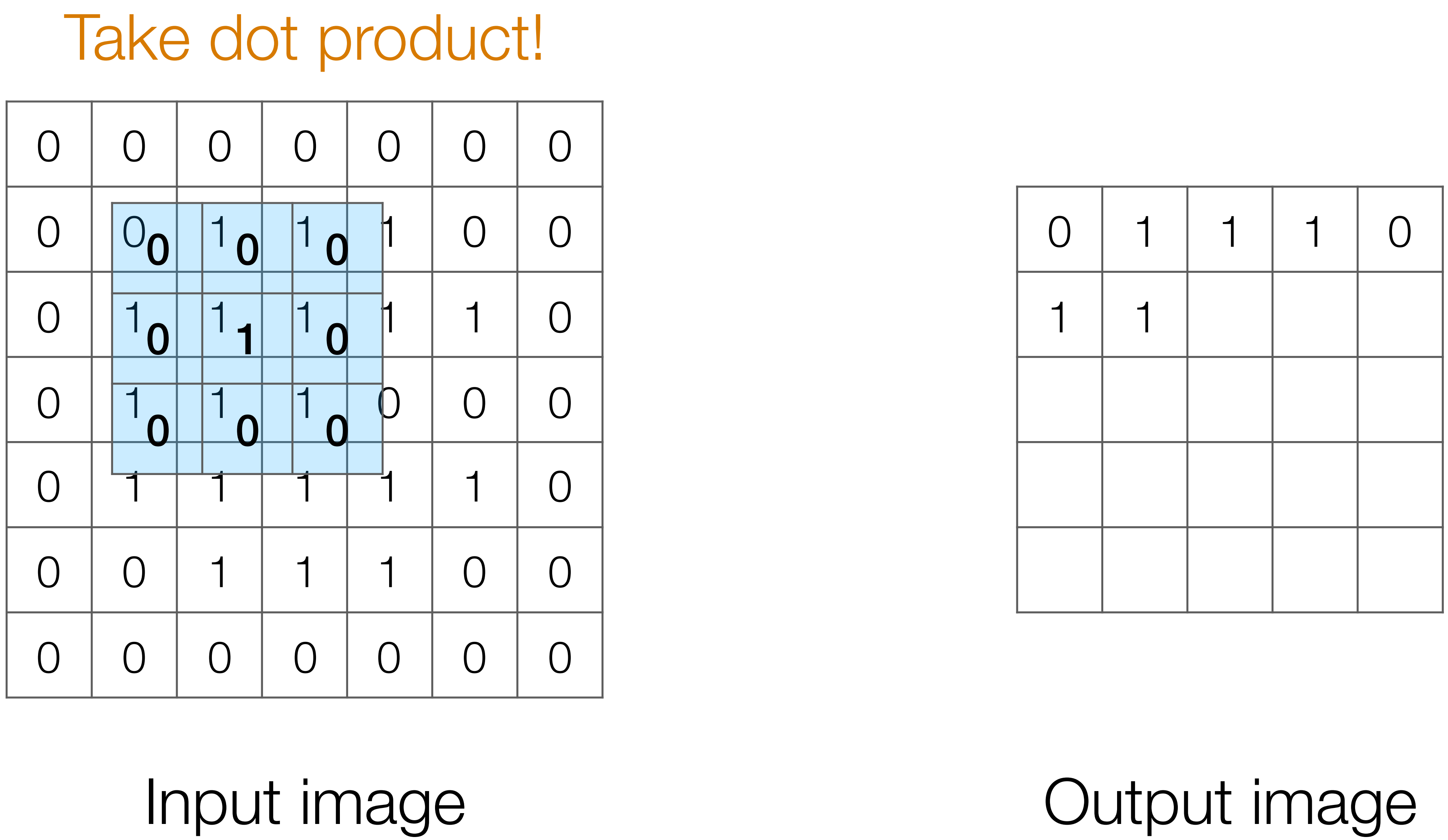
Convolution



Input image

Output image

Convolution



Convolution

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
*
\begin{bmatrix}
\mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{1} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0}
\end{bmatrix}
=
\begin{bmatrix}
0 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 0
\end{bmatrix}
$$

Input image

Output image

Note: output image is smaller than input image

If you want output size to be same as input, pad 0’s to input

Convolution

$$
\begin{bmatrix}
0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0\\
0&0&0&1&1&1&0&0&0\\
0&0&1&1&1&1&1&0&0\\
0&0&1&1&1&0&0&0&0\\
0&0&1&1&1&1&1&0&0\\
0&0&0&1&1&1&0&0&0\\
0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0
\end{bmatrix}
*
\begin{bmatrix}
0&0&0\\
0&1&0\\
0&0&0
\end{bmatrix}
=
\begin{bmatrix}
0&0&0&0&0&0&0\\
0&0&1&1&1&0&0\\
0&1&1&1&1&1&0\\
0&1&1&1&0&0&0\\
0&1&1&1&1&1&0\\
0&0&1&1&1&0&0\\
0&0&0&0&0&0&0
\end{bmatrix}
$$

Input image

Output image

Note: output image is smaller than input image

If you want output size to be same as input, pad 0's to input

Convolution

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Input image

Output image

Convolution

$$\begin{bmatrix} 0&0&0&0&0&0&0 \\ 0&0&1&1&1&0&0 \\ 0&1&1&1&1&1&0 \\ 0&1&1&1&0&0&0 \\ 0&1&1&1&1&1&0 \\ 0&0&1&1&1&0&0 \\ 0&0&0&0&0&0&0 \end{bmatrix} * \frac{1}{9}\begin{bmatrix} 1&1&1 \\ 1&1&1 \\ 1&1&1 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 3&5&6&5&3 \\ 5&8&8&6&3 \\ 6&9&8&7&4 \\ 5&8&8&6&3 \\ 3&5&6&5&3 \end{bmatrix}$$

Input image

Output image

Convolution

0	0	0	0	0	0	0
0	0	1	1	1	0	0
0	1	1	1	1	1	0
0	1	1	1	0	0	0
0	1	1	1	1	1	0
0	0	1	1	1	0	0
0	0	0	0	0	0	0

*

-1	**-1**	**-1**
2	**2**	**2**
-1	**-1**	**-1**

=

0	1	3	1	0
1	1	1	3	3
0	0	-2	-4	-4
1	1	1	3	3
0	1	3	1	0

Input image

Output image

Convolution

Very commonly used for:

- Blurring an image

*

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

=

- Finding edges

*

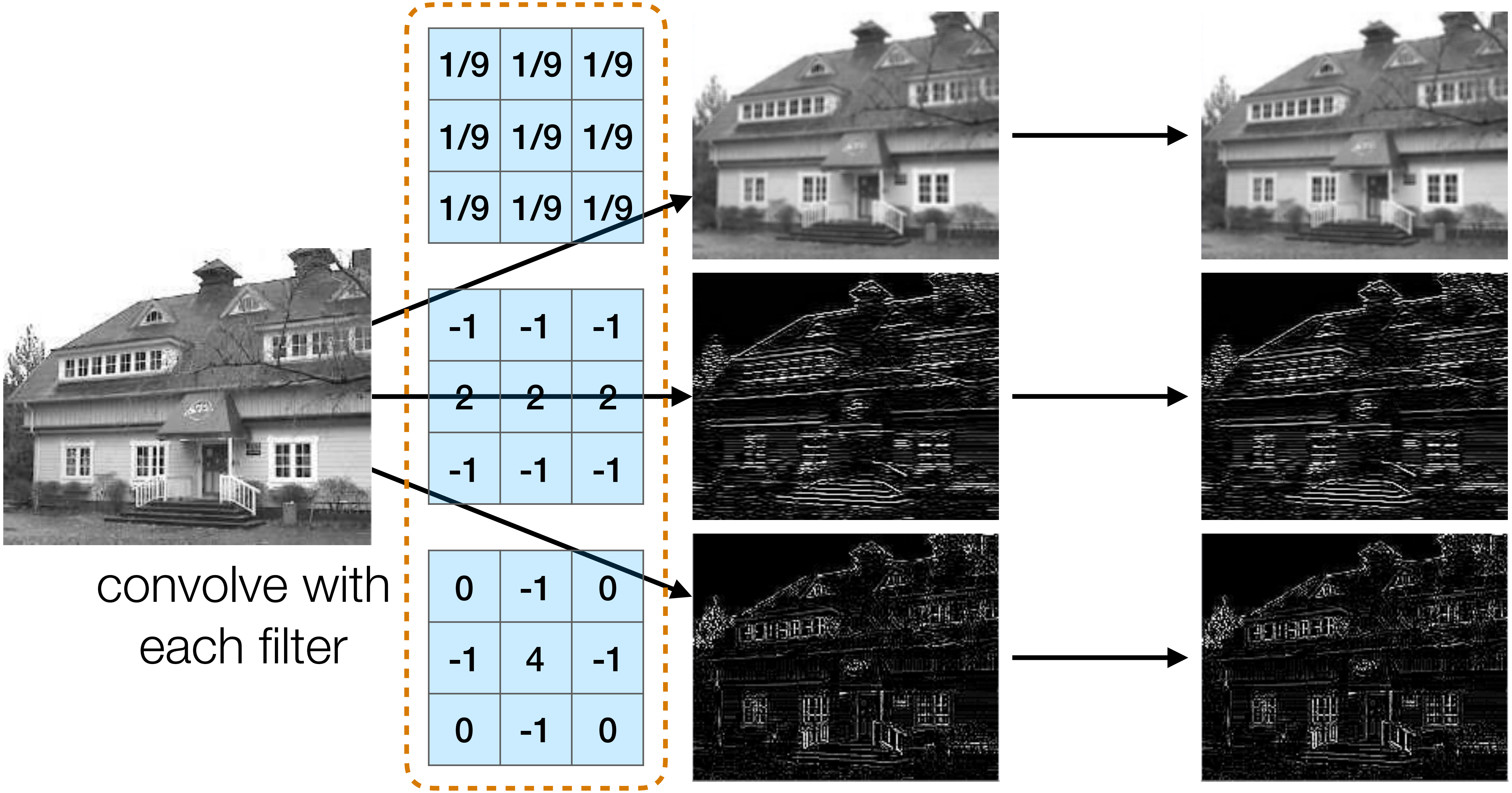
-1	-1	-1
2	2	2
-1	-1	-1

=

(this example finds horizontal edges)

Images from: http://aishack.in/tutorials/image-convolution-examples/

Convolution Layer

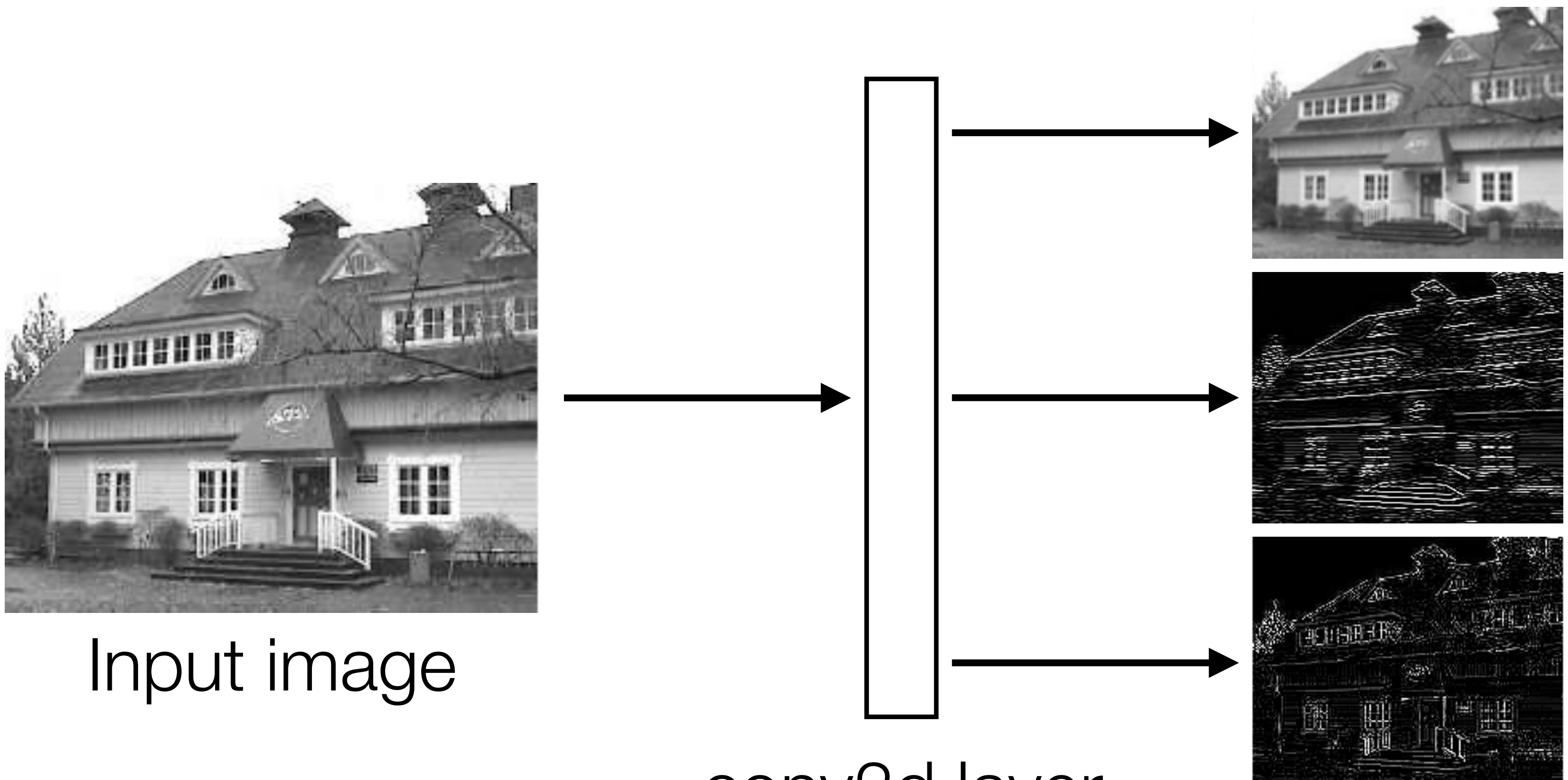


filters are actually unknown
and are learned!

activation (e.g., ReLU)

Images from: http://aishack.in/tutorials/image-convolution-examples/

Convolution Layer



Input image

conv2d layer
with ReLu activation
and three 3x3 kernels

Output images

Images from: http://aishack.in/tutorials/image-convolution-examples/

Convolution Layer

Input image

dimensions:
height,
width

conv2d layer
with ReLu activation
and three 3x3 kernels

Stack output images into a single “output feature map”

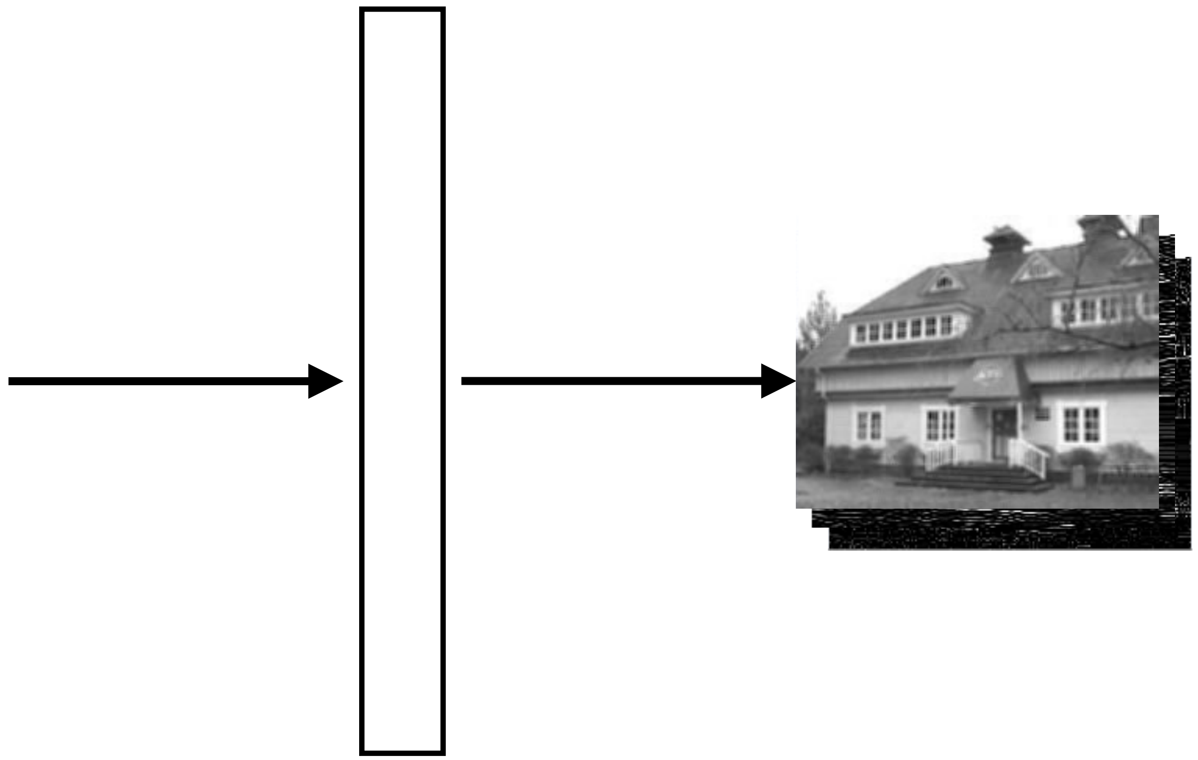
dimensions:
height-2,
width-2,
number of kernels
(3 in this case)

Images from: http://aishack.in/tutorials/image-convolution-examples/

Convolution Layer

Input image

dimensions:
height,
width



conv2d layer
with ReLu activation
and *k* 3x3 kernels

Stack output images into a single "output feature map"

dimensions:
height-2,
width-2,
k

Images from: http://aishack.in/tutorials/image-convolution-examples/

Convolution Layer

Input image

dimensions:
height,
width,
depth *d* (# channels)

conv2d layer
with ReLu activation
and *k* 3x3x*d* kernels

technical detail: there's also a bias vector

Stack output images into a single "output feature map"

dimensions:
height-2,
width-2,
k

Images from: http://aishack.in/tutorials/image-convolution-examples/

Pooling

- Aggregate local information

- Produces a smaller image
  (each resulting pixel captures some “global” information)

- If object in input image shifts a little, output is the same

Max Pooling

0	0	0	0	0	0	0
0	0	1	1	1	0	0
0	1	1	1	1	1	0
0	1	1	1	0	0	0
0	1	1	1	1	1	0
0	0	1	1	1	0	0
0	0	0	0	0	0	0

Input image

*

-1	**-1**	**-1**
2	**2**	**2**
-1	**-1**	**-1**

=

0	1	3	1	0
1	1	1	3	3
0	0	-2	-4	-4
1	1	1	3	3
0	1	3	1	0

Max Pooling

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} \mathbf{-1} & \mathbf{-1} & \mathbf{-1} \\ \mathbf{2} & \mathbf{2} & \mathbf{2} \\ \mathbf{-1} & \mathbf{-1} & \mathbf{-1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 3 & 1 & 0 \\ 1 & 1 & 1 & 3 & 3 \\ 0 & 0 & -2 & -4 & -4 \\ 1 & 1 & 1 & 3 & 3 \\ 0 & 1 & 3 & 1 & 0 \end{bmatrix}$$

Input image

$$\begin{bmatrix} \boxed{0} & \boxed{1} & 3 & 1 & 0 \\ \boxed{1} & \boxed{1} & 1 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 3 & 3 \\ 0 & 1 & 3 & 1 & 0 \end{bmatrix}$$

Output image after ReLU

$$\begin{bmatrix} & \\ & \end{bmatrix}$$

Output after max pooling

Max Pooling

$$\begin{bmatrix} 0&0&0&0&0&0&0 \\ 0&0&1&1&1&0&0 \\ 0&1&1&1&1&1&0 \\ 0&1&1&1&0&0&0 \\ 0&1&1&1&1&1&0 \\ 0&0&1&1&1&0&0 \\ 0&0&0&0&0&0&0 \end{bmatrix} * \begin{bmatrix} \mathbf{-1}&\mathbf{-1}&\mathbf{-1} \\ \mathbf{2}&\mathbf{2}&\mathbf{2} \\ \mathbf{-1}&\mathbf{-1}&\mathbf{-1} \end{bmatrix} = \begin{bmatrix} 0&1&3&1&0 \\ 1&1&1&3&3 \\ 0&0&-2&-4&-4 \\ 1&1&1&3&3 \\ 0&1&3&1&0 \end{bmatrix}$$

Input image

0	1	3	1	0
1	1	1	3	3
0	0	0	0	0
1	1	1	3	3
0	1	3	1	0

Output image after ReLU

1	

Output after max pooling

Max Pooling

0	0	0	0	0	0	0
0	0	1	1	1	0	0
0	1	1	1	1	1	0
0	1	1	1	0	0	0
0	1	1	1	1	1	0
0	0	1	1	1	0	0
0	0	0	0	0	0	0

Input image

*

-1	**-1**	**-1**
2	**2**	**2**
-1	**-1**	**-1**

=

0	1	3	1	0
1	1	1	3	3
0	0	-2	-4	-4
1	1	1	3	3
0	1	3	1	0

0	1	3	1	0
1	1	1	3	3
0	0	0	0	0
1	1	1	3	3
0	1	3	1	0

Output image after ReLU

1	3

Output after max pooling

Max Pooling

0	0	0	0	0	0	0
0	0	1	1	1	0	0
0	1	1	1	1	1	0
0	1	1	1	0	0	0
0	1	1	1	1	1	0
0	0	1	1	1	0	0
0	0	0	0	0	0	0

Input image

*

-1	**-1**	**-1**
2	**2**	**2**
-1	**-1**	**-1**

=

0	1	3	1	0
1	1	1	3	3
0	0	-2	-4	-4
1	1	1	3	3
0	1	3	1	0

0	1	3	1	0
1	1	1	3	3
0	0	0	0	0
1	1	1	3	3
0	1	3	1	0

Output image after ReLU

1	3
1	

Output after max pooling

Max Pooling

0	0	0	0	0	0	0
0	0	1	1	1	0	0
0	1	1	1	1	1	0
0	1	1	1	0	0	0
0	1	1	1	1	1	0
0	0	1	1	1	0	0
0	0	0	0	0	0	0

Input image

*

-1	**-1**	**-1**
2	**2**	**2**
-1	**-1**	**-1**

=

0	1	3	1	0
1	1	1	3	3
0	0	-2	-4	-4
1	1	1	3	3
0	1	3	1	0

0	1	3	1	0
1	1	1	3	3
0	0	0	0	0
1	1	1	3	3
0	1	3	1	0

Output image after ReLU

1	3
1	3

Output after max pooling

Max Pooling

0	0	0	0	0	0	0
0	0	1	1	1	0	0
0	1	1	1	1	1	0
0	1	1	1	0	0	0
0	1	1	1	1	1	0
0	0	1	1	1	0	0
0	0	0	0	0	0	0

Input image

*

-1	**-1**	**-1**
2	**2**	**2**
-1	**-1**	**-1**

=

0	1	3	1	0
1	1	1	3	3
0	0	-2	-4	-4
1	1	1	3	3
0	1	3	1	0

0	1	3	1	0
1	1	1	3	3
0	0	0	0	0
1	1	1	3	3
0	1	3	1	0

Output image after ReLU

What numbers were involved in computing this 1?

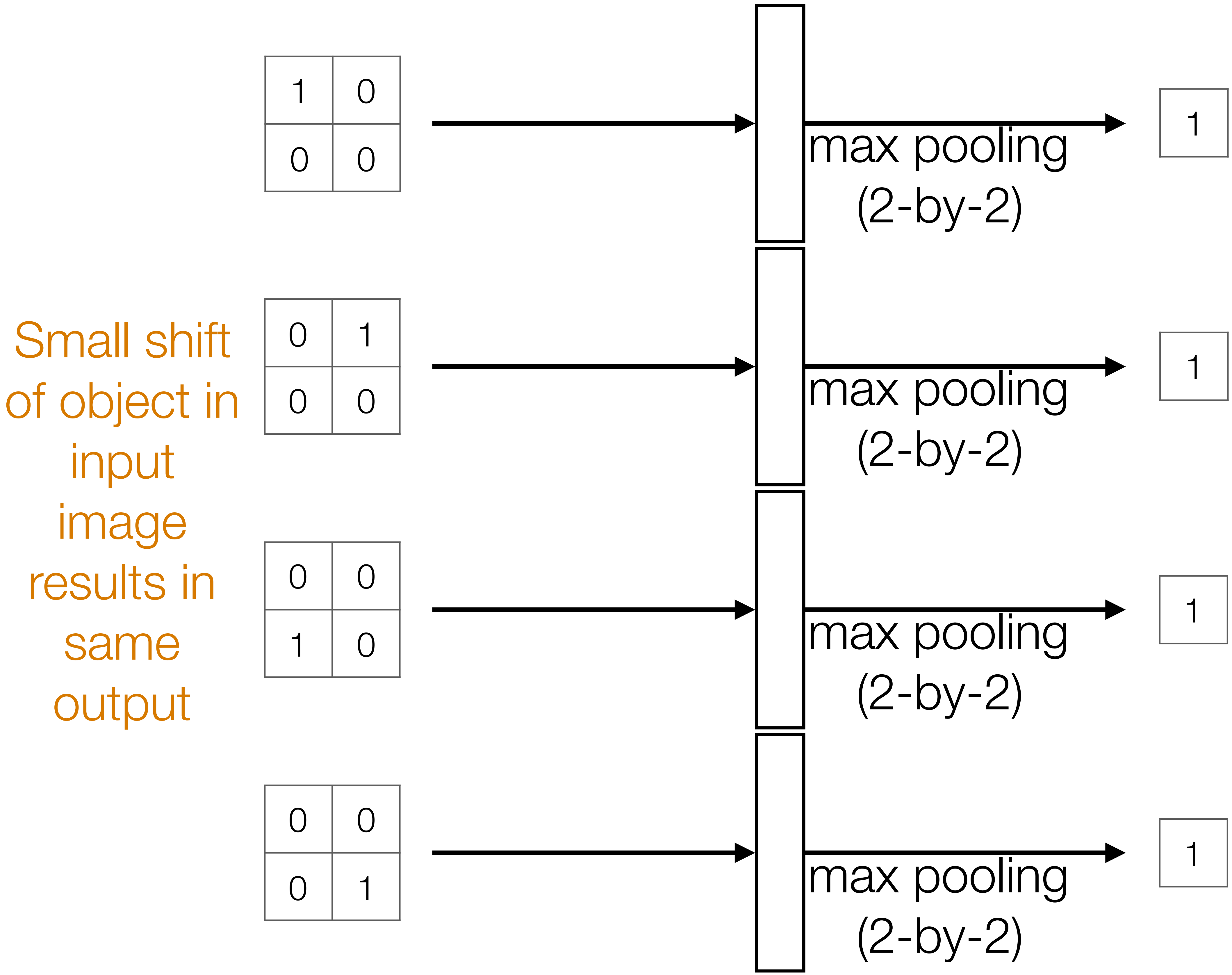
1	3
1	3

Output after max pooling

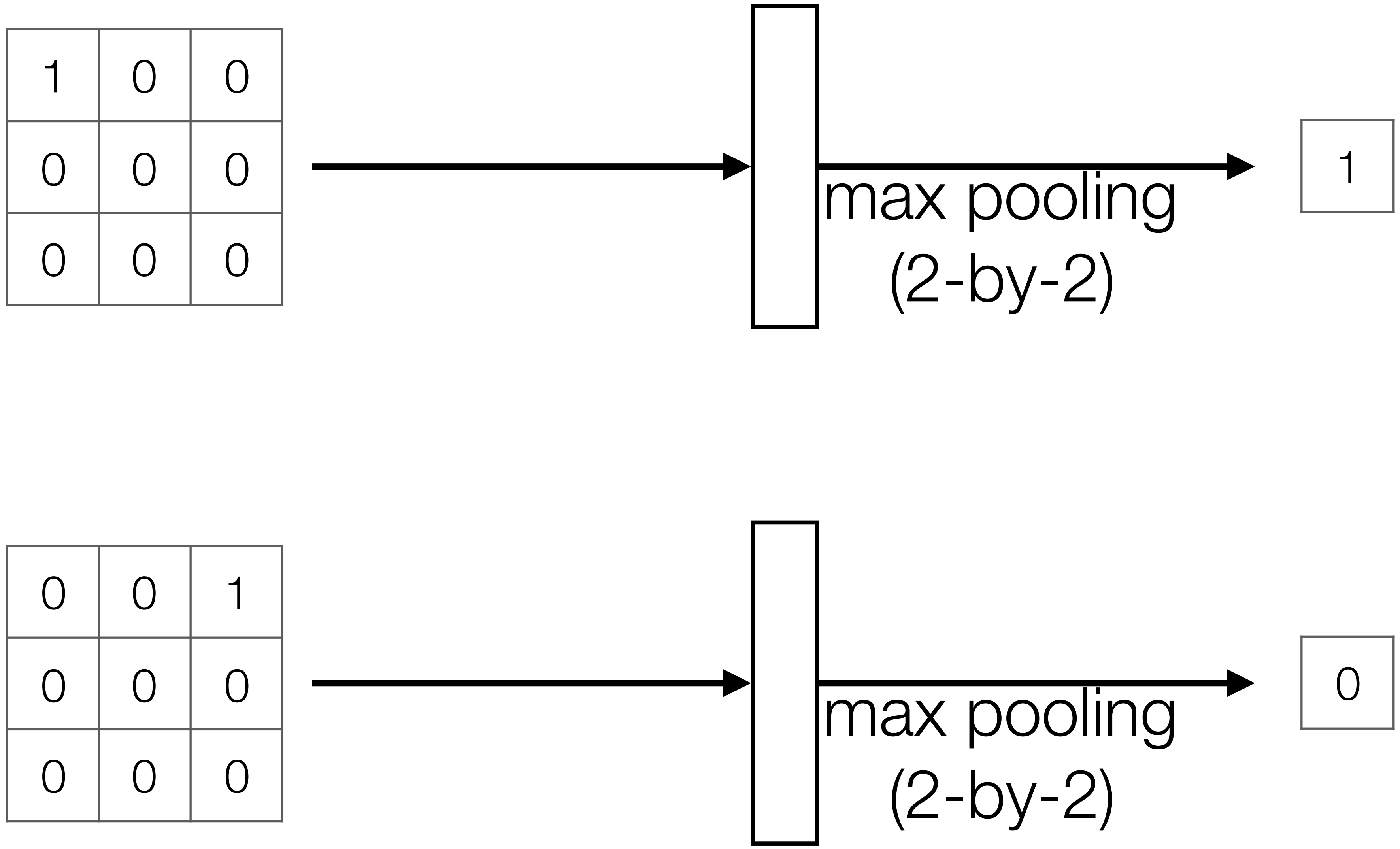
In this example: 1 pixel in max pooling output captures information from 16 input pixels!

Example: applying max pooling again results in a single pixel that captures info from entire input image!

Max Pooling and (Slight) Shift Invariance

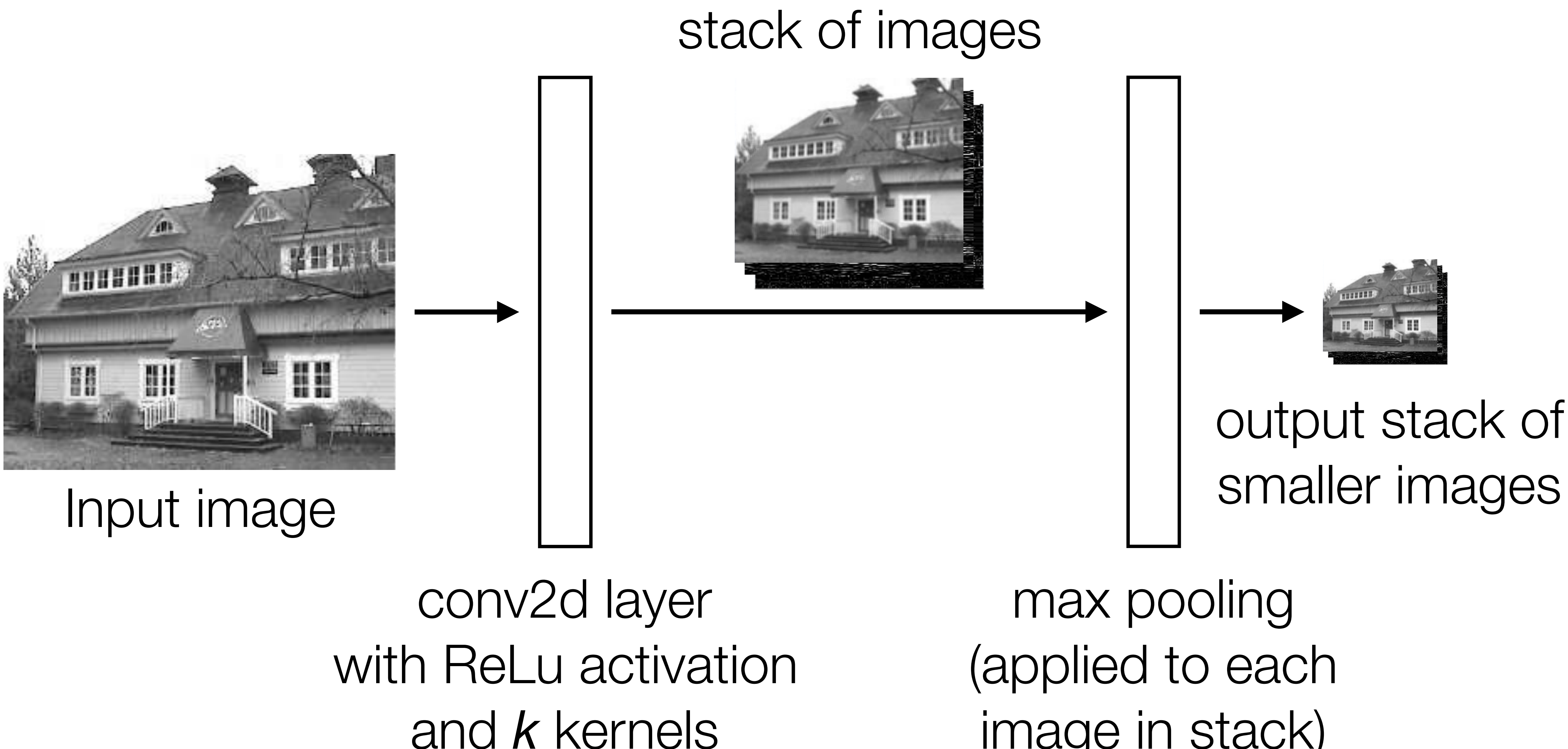


Max Pooling and (Slight) Shift Invariance



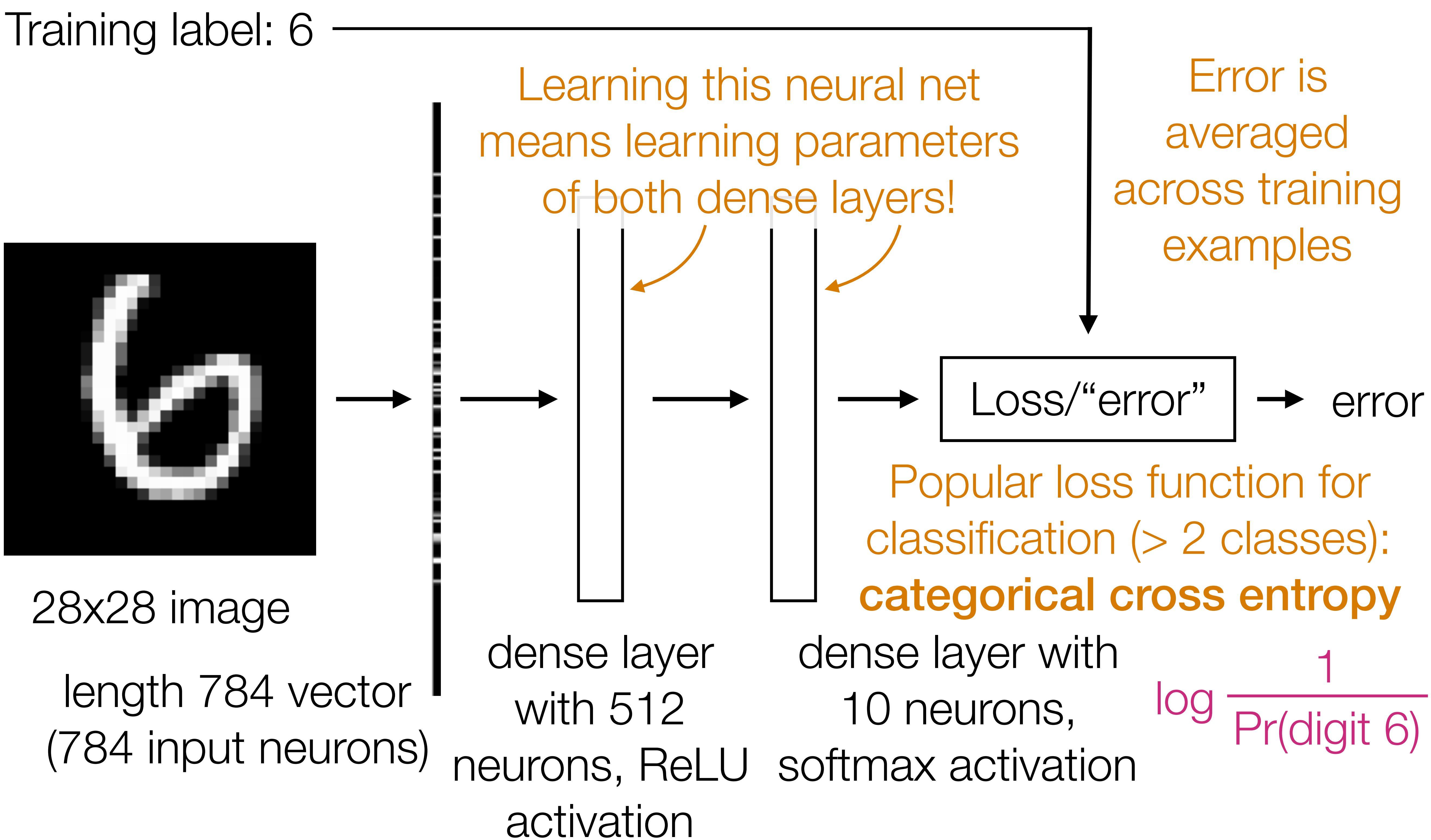
Big shift in input can still change output

Basic Building Block of CNN’s

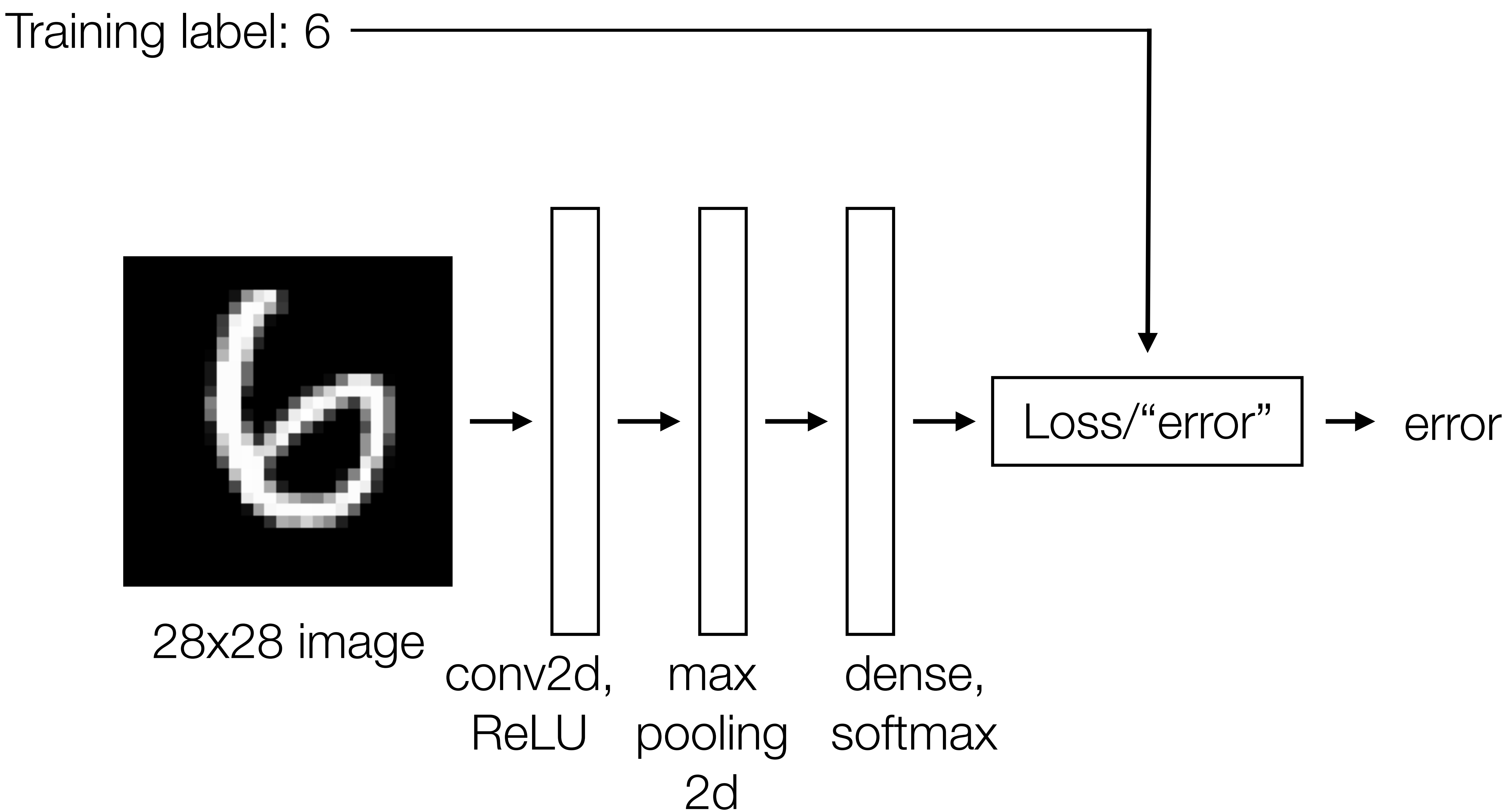


Images from: http://aishack.in/tutorials/image-convolution-examples/

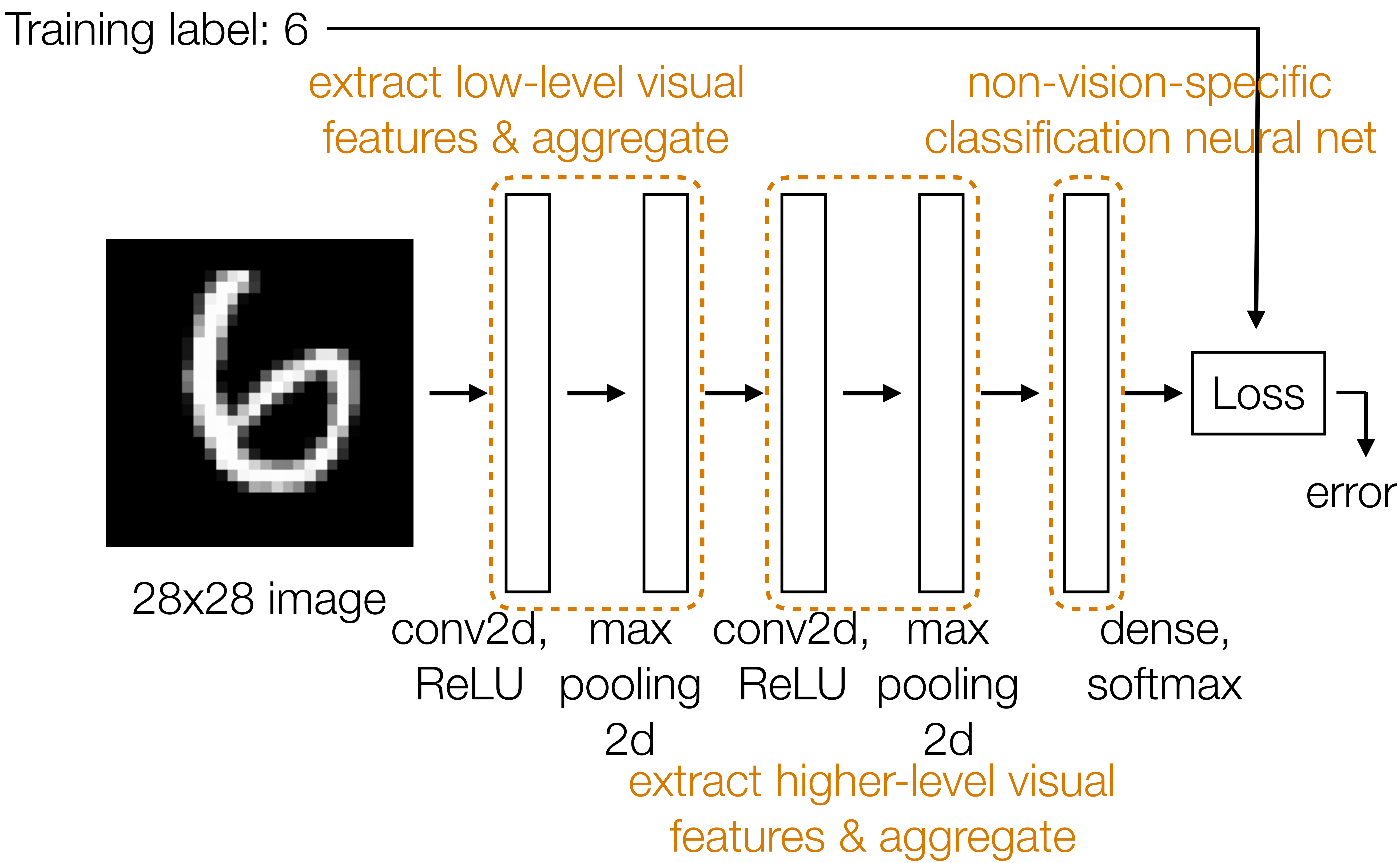
Handwritten Digit Recognition



Handwritten Digit Recognition



Handwritten Digit Recognition



CNN Demo

CNN’s

- Learn convolution filters for extracting simple features

- Max pooling summarizes information and produces a *smaller* output and is invariant to small shifts in input objects

- Can then repeat the above two layers to learn features from increasingly higher-level representations